

Math 177. Geometric Methods in ODE
SPRING 2015

Take-home Midterm Exam

Due on Monday, May 11

1. A particle of mass m moves on the surface of the cone

$$x^2 + y^2 - cz^2 = 0, \quad z \leq 0, \quad c > 0,$$

in the constant field of gravity g parallel to the axis of the cone. Integrate the equations of motion and describe the trajectories.

2. Prove that a curve which is projectively dual to an ellipse is again an ellipse.

3. In \mathbb{R}^3 with coordinates (x, y, z) consider a differential one form $\alpha = dz - ydx$. Given any function $K : \mathbb{R}^3 \rightarrow \mathbb{R}$, show that there exists a unique vector field \mathbf{v} which satisfies the following two conditions:

- $\alpha(\mathbf{v}) = K$;
- $L_{\mathbf{v}}\alpha = c\alpha$ for some function $c : \mathbb{R}^3 \rightarrow \mathbb{R}$,

and find explicitly this vector field.

4. Prove that the following Cauchy problem :

$$(x^3 - 3xy^2)\frac{\partial u}{\partial x} + (3x^2y - y^3)\frac{\partial u}{\partial y} = 0, \quad u|_{x^2+y^2=2} = \sin y,$$

has no solution in a neighborhood of the point $(1, 1)$.

Each problem, including subproblems in 2 is 10 points.