Math 177. Geometric Methods in ODE

Spring 2015

Take-home Final Exam

Due on Wednesday, June 3

1. Find explicitly action – angle variables for the mathematical pendulum:

$$H = \frac{1}{2}p^2 - \cos q.$$

2. Consider the contact plane field $\xi = \{dz - ydz = 0\}$ in $\mathbb{R}^3$. A 1 dimensional submanifold $\Gamma \subset \mathbb{R}^3$ is called Legendrian if it is tangent to ξ. Denote by π the projection $(x, y, z) \mapsto (x, y)$. Suppose that the submanifold Γ is connected and closed (i.e. diffeomorphic to a circle). Prove that the projected curve $\pi(\Gamma) \subset \mathbb{R}^2$ must have self intersection points.

3. Consider a Lagrangian system in the upper half plane

$$\{(x, y);\ y > 0\} \subset \mathbb{R}^2$$

with the Lagrangian function

$$L(x, y, \dot{x}, \dot{y}) = \frac{\dot{x}^2 + \dot{y}^2}{y^2}.$$

Write the equation of motion in the Hamiltonian form, find explicitly all the trajectories and describe qualitatively their behavior.

Each problem is 10 points.