Serre Seminar

Goal: Come to as complete an understanding of Serre's conjecture and Khare's work (see his homepage) and its applications as time allows.

Topics: The topics to be covered, listed in order and with the associated references, are the following. References are indicated when known; undoubtedly more will arise as we proceed.

- Compatible families of p-adic Galois representations (examples, basic properties, characterization via characteristic polynomial in irreducible case). Reference: Serre's book on abelian ℓ-adic representations.
- Preliminary statement of Serre's conjecture, relation to 2-dimensional Artin conjecture.
- p-adic Galois representations coming from abelian varieties and Hilbert modular forms, local properties "away from p". (Skinner)
- finite group schemes and p-divisible groups: theorem of Raynaud, relation with peu ramifie condition, ordinary and connected p-divisible groups, Barsotti–Tate property of p-adic Galois representations.
- Precise statement of (and motivation for) Serre's conjecture, both weak and strong forms. Reference: Serre's 1987 Duke paper "Sur les représentations modulaires …". (Cais)
- Tate's proof of the conjecture for level 1 and $p = 2$, case of Artin representations with solvable image (Langlands-Tunnell). (Grigorov)
- p-adic Galois representations "at p" and their properties: ordinary, Barsotti–Tate, crystalline, and potential versions. General philosophy behind (and description of) Fontaine's period rings. Main theorems of Kisin and Skinner–Wiles as used by Khare. (Conrad)
- Outline of Khare's paper. (Skinner)
- Basics of deformation theory of Galois representations: generalities on deformations and Galois cohomology, especially existence criteria and tangent spaces (References: Mazur's article in the FLT book, Serre's Galois Cohomology book), deformation conditions, arithmetic of Galois cohomology (Tate's theorems: local/global duality and local/global Euler characteristics). (Arnold/Klosin)
- Ramakrishna's lifting theorem: Theorem 1b in Annals 156 (no. 1), 2002, pp. 115-154. (Agarwal)
- Arithmetic of abelian varieties and Taylor's theorem on torsion in abelian varieties (Theorem F in his preprint "Remarks on a conjecture of Fontaine–Mazur").
- Weak Serre implies strong Serre: overview of results, arithmetic of modular curves, classical modular forms, and Hecke rings, Ribet's level-lowering (1990 Inventiones paper), Edixhoven's weight-lowering (1992 Inventiones paper).

These last three items are rather substantial, and will likely take up most of the effort of the year-long seminar. Aside from the key inputs from modularity theorems (of Kisin and Skinner–Wiles, which we shall take as black boxes), these constitute the key technological input that make Khare's method work and they will provide us with lots of experience applying modern number theory, algebraic geometry, and commutative algebra to prove interesting concrete theorems.