Math 210C. Compact Lie groups

Instructor. Brian Conrad, 383CC Sloan Hall, `conrad@math.stanford.edu`

Course assistant. Jesse Silliman, 384M Sloan Hall, `silliman@stanford.edu`

Office hours. (Conrad) MW, 3:30-5pm (at tea during 3:30-4); (Silliman) TuTh, 4-5:30pm.

Prerequisites: Multilinear algebra and basic notions related to topology and smooth manifolds (e.g., submersion theorem, smooth vector fields, differential forms, integration of top-degree forms using orientation). It is extremely important to be quite comfortable with this background.

Occasionally we use 210AB algebra notions such as integrality, finite-degree Galois theory (in characteristic 0), affine varieties with the Zariski topology, and Ext-functors. If you don't know (or are rusty on) some of this algebra material then you can still follow nearly the entire course.

Textbooks: *Representations of Compact Lie Groups* by Bröcker and tom Dieck

Homework/exams: For undergraduates registered for this class, homework is due *every* Friday *at the start of class*, and is posted at the course webpage. The first homework will be posted on the course webpage before the first class and it is due on the first Friday of the course (it only requires material from the first lecture and assumed background in algebra and topology).

Some important topics and examples, such as covering space theory and some aspects of root systems, will be developed *entirely in the Homeworks* before being used in the class. Thus, pay vigilant attention to the homework, even if you're a graduate student not turning it in.

There will be no exams; the final grade (for undergraduates) is 100% homework. You may certainly work with others on the homework (and are encouraged to discuss the material with classmates), but please write up solutions on your own.

Late homework is not accepted for any reason whatsoever. However, the lowest homework grade is dropped. (A late homework counts as a zero.) Be careful not to "waste" this option by using it up too early in the quarter!

Course description: This is the third quarter of the year-long sequence in algebra at the graduate student level, focusing on compact Lie groups. The definition is simple: a compact C^∞ manifold G equipped with a group structure for which the composition law $G \times G \to G$ and inversion are C^∞ maps. There are a lot of classical examples arising from the study of matrix groups, as we will see. The thoroughly "analytic" nature of the initial definition makes it all the more amazing that there is an underlying "algebraicity" in the theory (via the prominent role of matrix groups). One aim of the course is to explain that miracle.

Our main goal is to cover the structure theory of connected compact semisimple Lie groups, an excellent "test case" for the general theory of connected semisimple Lie groups (avoiding many analytic difficulties, thanks to compactness). This includes a tour through the "classical (compact) groups", the remarkable properties of maximal tori (especially conjugacy thereof), some basic ideas related to Lie algebras, and the combinatorial concepts of root datum and Weyl group which lead to both a general classification in the compact connected case as well as a foothold into the representation theory of semisimple compact connected Lie groups.

These topics arise in an astonishing array of situations in representation theory, algebraic geometry, differential geometry, number theory, and physics. We will largely avoid getting into the precise structure theory of semisimple Lie algebras, but experience with Lie groups as provided by this course gives much motivation for as well as intuition about Lie algebras.

This course moves rapidly, so please don't let yourself fall far behind!