ADDENDUM TO 'FIXED ENERGY INVERSE PROBLEM FOR EXPONENTIALLY DECREASING POTENTIALS'

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The main result of this paper was a new proof of the fixed energy inverse result for exponentially decaying potentials in potential scattering, which was first obtained by Novikov [4], see also the work of Novikov and Khenkin [3]. We used two main techniques: real analyticity in part of the complex momentum (Proposition 2.2) and a density argument. The former had been noted in [3], as we pointed out in the paper. We provided a simple proof by showing that the appropriate quantities have an analytic continuation (over which we needed no control) – this is similar to the analytic continuation arguments for the resolvent of the Laplacian as can be found, for instance, in Melrose's book [2, pp. 7-9].

What we did not realize that after Novikov, but several years before we started working on this project, Eskin and Ralston wrote a paper [1] in which they not only extended Novikov's result to the magnetic setting, but also simplified his proof in the potential setting. Unfortunately we were unaware of this aspect of their paper, as well as of their proofs. It turns out that the contour deformation proof we gave for real analyticity is substantially identical to that of Eskin and Ralston [1, pp. 204-205], although they also provide more control over the analytic continuation, and use it elsewhere in their paper. We very much regret to have omitted the reference to [1].

The rest of the proof of the inverse result is different; ours relies on a density argument, while that of Eskin and Ralston on taking a limit to real frequencies.

References

[1] G. Eskin and J. Ralston. Inverse scattering problem for the Schrödinger equation with magnetic potential at a fixed energy. *Commun. Math. Phys.*, 173:199–224, 1995.

[2] R. B. Melrose. *Geometric scattering theory.* Cambridge University Press, 1995.

[3] R. G. Novikov and G. M. Khenkin. The $\overline{\partial}$-equation in the multidimensional inverse scattering problem. *Uspekhi Mat. Nauk*, 42(3(255)):93–152, 255, 1987.

[4] Roman G. Novikov. The inverse scattering problem at fixed energy for the three-dimensional Schrödinger equation with an exponentially decreasing potential. *Comm. Math. Phys.*, 161(3):569–595, 1994.

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