

MATH 131P: PROBLEM SET 8
DUE WEDNESDAY, NOVEMBER 28, 2012

Do the following problems from the textbook: Lesson 24:3,4,5, Lesson 25:1,3, Lesson 26:1,2,4, Lesson:27:1,2,3,5, as well as the following problem:

Problem 1. For $c > 0$ constant, $\phi = \phi(\theta)$ and $\psi = \psi(\theta)$ given, consider the wave equation

$$u_{tt} = c^2 u_{\theta\theta}$$

on a ring, parameterized by the angle θ in the circle, so $\theta \in [0, 2\pi]$, $t \geq 0$, and the points $\theta = 0$ and $\theta = 2\pi$ are considered the same, and thus we impose the so-called periodic boundary conditions

$$\begin{aligned} u(0, t) &= u(2\pi, t), \\ u_\theta(0, t) &= u_\theta(2\pi, t). \end{aligned}$$

Assume that the initial position and velocity are given by

$$u(\theta, 0) = \phi(\theta), \quad u_t(\theta, 0) = \psi(\theta).$$

Using separation of variables, find u in this generality, leaving the coefficients as integrals (but solve the ODEs).

Further, solve the problem in a different way by considering ϕ, ψ as 2π -periodic functions on \mathbb{R} (so $\phi(\theta + 2\pi) = \phi(\theta)$ for all $\theta \in \mathbb{R}$), and use d'Alembert's formula on $\mathbb{R}_\theta \times [0, \infty)_t$.

Where do you need to know the initial data ϕ and ψ to find u_t and u_θ ? Can you describe where u_t and u_θ are C^k (k -times continuously differentiable) in terms of where the initial data satisfy $\phi \in C^{k+1}$ and $\psi \in C^k$?