MATH 131P: PROBLEM SET 6
DUE WEDNESDAY, NOVEMBER 7, 2012

Do the following problems from the textbook: Lesson 17: 1,3,4, Lesson 18.1,3, Lesson 19.1,2, as well as the following problem:

Problem 1. Calculate the Fourier transform of the function $f(x) = e^{-ax^2}$, $a > 0$, as follows. First rewrite the integral

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-ix\xi} e^{-ax^2}\, dx = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-(ax^2+ix\xi)}\, dx$$

by completing the square in x. Pull out a Gaussian factor in ξ, i.e. $e^{-b\xi^2}$ for some $b > 0$, so that you are left with an integral of the form

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-a(x+i\alpha\xi)^2}\, dx$$

for some α. Then show that this is independent of ξ by showing that its ξ derivative is 0: when you differentiate, do it under the integral sign, and rewrite to have an expression (still in the integrand) that is of the form $\frac{\partial g}{\partial x}$ for some g which decays as $|x| \to \infty$, so by the fundamental theorem of calculus the integral (in x!) is 0. Finally, evaluate

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-a(x+i\alpha\xi)^2}\, dx$$

when $\xi = 0$: there is a standard trick in calculus for doing this. (Take the square of this expression, writing the two factors as integrals in different variables x and y, and convert to polar coordinates.)