18.725: INTRODUCTION TO ALGEBRAIC GEOMETRY

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Algebraic geometry has links to many other fields of mathematics: number theory, differential geometry, topology, mathematical physics, and more.

How to reach me. The best thing to do is to drop by my office (2-271) or e-mail me (vakil@math.mit.edu). The phone number in my office is 3-2683.

Office hours. To be announced. The course will be small, so I may just have office hours by appointment. In any case, I'm around MIT most days, so please drop by any time — the subject is best picked up by asking questions and talking it through, not by watching someone do it at a blackboard.

Homework. Algebraic geometry can be very abstract, and the only real way to learn it is to see how you can actually compute things. If you're taking the course for credit, the only component of the grade will be problem sets. Even if you're not taking the course for a grade (but simply to learn), you should definitely at least try the homework problems. The grader will almost definitely be Ana-Maria Castravet. The problem sets will be roughly weekly, and there will be roughy ten of them.

References. We won't follow any one reference too closely, but it will be useful to refer to the following three (in order of relevance). I think/hope they're all in the Coop.

1. Mumford's *Red Book of Varieties and Schemes*, Lecture Notes in Math Vol. 1358.
2. Shafarevich's *Basic Algebraic Geometry 1: Varieties in Projective Space*, 2nd ed.
3. Hartshorne's *Algebraic Geometry* Chapter 1. (Hartshorne is a canonical reference, but sometimes an intimidating place to learn the material.)

Rarely you might want to refer to a book in commutative book. Lang's *Algebra* is good, as is Eisenbud's *Commutative Algebra with a View Toward Algebraic Geometry*. Matsumura's *Commutative Ring Theory* is a hard-core reference.

Tentative sampling of topics.

Varieties.

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Introduction. Algebraic sets. Hilbert's Nullstellensatz. Noetherian rings and the Hilbert basis theorem.

Affine varieties. Projective varieties, and varieties in general. Properties, morphisms, products, etc.

Local properties. Dimension.

Sheaves.

Sheaves and vector bundles.

Divisors (on smooth varieties) and invertible sheaves (= line bundles); maps to projective space.

Applications of the Riemann-Roch theorem.

An example we'll follow throughout. Complex curves (and in general curves over any algebraically closed field).

Other examples. (a) Projective space $\mathbb{P}^n$; $\mathbb{P}^1 \times \mathbb{P}^1$. (b) $\mathbb{Z}$; finite extensions of $\mathbb{Z}$.

Topics we won't discuss: quasicoherent sheaves, cohomology, differentials, flatness. I hope to introduce the idea of a *scheme* to give you some exposure to the concept, but not to use it in any essential way. (The follow-up course, taught by Johan de Jong in the spring, will be all about schemes.)