MATH 121 PROBLEM SET 7

This set is due at noon on Friday March 16 in Jason Lo's mailbox.

1. Work through how to solve a general cubic by solving the specific cubic

$$t^3 - 7t + 6 = 0.$$

Do *not* just guess roots, or use the formula; work through each step!

2. Work through how to solve a general quartic by solving the specific quartic

$$t^4 - 15t^2 + 10t + 24 = 0.$$

Do *not* just guess roots, or use the formula; work through each step, except that you can just state the solutions to the resolvent cubic.

3. (You may find it helpful to read the brief commentary on the Galois group of the quartic in §14.6.) In the last problem set, you computed the minimal polynomial for $\sqrt{2+\sqrt{2}}$ over $\mathbb{Q}$, and showed that its splitting field had Galois group (over $\mathbb{Q}$) cyclic of order 4. If you were given this polynomial, and were told that it was irreducible, in the course of finding its roots, there would have been clues that would also prove that the Galois group was $\mathbb{Z}/4$. Explain how. For example, the resolvent cubic factors into a linear term times a quadratic term.

4. In earlier work, you computed the minimal polynomial for $\sqrt{2}+\sqrt{3}$ over $\mathbb{Q}$ and showed that its splitting field had Galois group (over $\mathbb{Q}$) isomorphic to $\mathbb{Z}/2 \times \mathbb{Z}/2$. If you were given this polynomial, and were told that it was irreducible, in the course of finding its roots, there would have been clues that would also prove that the Galois group was $\mathbb{Z}/2 \times \mathbb{Z}/2$. Explain how. En route, you may show that if the Galois group of an irreducible quartic were $\mathbb{Z}/2 \times \mathbb{Z}/2$ (or more generally, contained in A_4), then its discriminant is a perfect square.

5. Suppose $q(x)$ is a quartic, and $p(x)$ is the resolvent cubic. Show that the discriminant of the quartic is a constant multiple of the discriminant of the cubic, and find that multiple. (Hint: do this with as little work as possible!)

Date: Friday, March 9, 2007.