MATH 121 PROBLEM SET 3

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This set is due at noon on Friday February 16 in Jason Lo's mailbox.

1. Write up a solution to the problem on the midterm on which you scored the least. (If there was a tie, pick the one you think is harder. If it was problem 1, then give full justifications.)

2. Suppose E/F is a finite extension, and F^{sep} is the intermediate field of separable elements of E (over F). Show that the degree of E/F^{sep} is a power of p. Show that any power of p is possible.

3. Prove that the automorphisms of the rational function field $k(t)$ which fix k are precisely the *fractional linear transformations* determined by $t \mapsto (at+b)/(ct+d)$ for $a, b, c, d \in k$, $ad - bc \neq 0$ (so $f(t) \in k(t)$ maps to $f((at+b)/(ct+d))$. (Dummit and Foote 14.1 problem 8)

4. *Different definitions of normality.* Suppose E/F is an algebraic field extension. Show that E/F is the splitting field of a family of polynomials if and only if every irreducible polynomial in $F[x]$ with a root in E splits completely. If E/F is finite, show that these two are equivalent to the statement that any map from E to $\overline{F}$ fixing F must send E to itself.

Date: Saturday, February 10, 2007. Corrected February 13.