MATH 121 PROBLEM SET 2

RAVI VAKIL

This set is due at noon on Friday February 2 in Jason Lo's mailbox.

1. How many cube roots of 1 are there in $\mathbb{F}_{p^n}$? (The answer will depend on p^n.)

2. Find the degree of the splitting field of $x^4 - 2$ over $\mathbb{Q}$.

3. Find the greatest common divisor of $x^{p^{12}} - x$ and $x^{p^8} - x$ (a) in $\mathbb{F}_p$, (b) in $\mathbb{F}_{p^2}$.

4. Find the degree of the compositum of $\mathbb{Q}(\sqrt[3]{3})$ and $\mathbb{Q}(\omega)$ in $\mathbb{C}$ (where ω is a primitive cube root of 1).

5. Find all irreducible polynomials of degrees 1, 2, and 4 over $\mathbb{F}_2$ and prove that their product is $x^{16} - x$. (Try to do this with as little work as possible!)

Date: Saturday, January 27, 2007.