

MATH 121 MIDTERM

RAVI VAKIL

Each problem is worth 6 points. Justify all answers!

1. No explanation is required for this problem.
 - (a) Which of { algebraic, finite, separable } implies which other (for field extensions)?
 - (b) Let $\overline{\mathbb{Q}}$ be the algebraic closure of \mathbb{Q} . Is it algebraic? Finite? Separable?
2. Prove that $\mathbb{Q}(\sqrt{5} + \sqrt{7}) = \mathbb{Q}(\sqrt{5}, \sqrt{7})$.
3. Suppose K is a field of characteristic p (not necessarily finite). Show that each element of K has at most one p th root. Give an example of such a field in which each element has a p th root. Give an example of such a field in which there exists an element without a p th root.
4. Show that the degree of the splitting field of a degree n polynomial is a factor of $n!$.
5. Find the group of automorphisms of the splitting field of $x^4 - 2$ over \mathbb{Q} .

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