18.034 PROBLEM SET 1

Due February 11 in class. No lates will be accepted. Discussion is encouraged, with two caveats: (a) write up your solutions by yourself, and (b) give credit when others came up with ideas (you won't be penalized for this). Give explanations, not just answers. References are to Boyce and DiPrima.

1. (p. 10 # 14) Verify that $y = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$ is a solution to the differential equation $y' - 2ty = 1$.
2. (a) (p. 10 # 20) Determine the values of r for which $t^2 y'' - 4ty' + 4y = 0$ has solutions of the form $y = t^r$ for $t > 0$.
   (b) Describe the general technique suggested by (a), that works for certain differential equations of any order. (Say precisely which differential equations you can solve by this method, and explain how to solve them.)
3. Find the solution of the following initial value problems:
   (a) (p. 23 # 18) $ty' + 2y = \sin t$, $y(\pi/2) = 1$.
   (b) (p. 23 # 20) $ty' + (t+1)y = t$, $y(\ln 2) = 1$.
4. Solve the following differential equations:
   (a) (p. 38 # 1) $y' = x^2/y$.
   (b) (p. 38 # 3) $y' + y^2 \sin x = 0$.
5. (p. 31 # 25) Find the value of y_0 for which the solution of the initial value problem
$$y' - y = 1 + 3\sin t,$$
   $y(0) = y_0$ remains finite (as $t \to \infty$).
6. (p. 39 # 22) Solve the initial value problem
$$y' = 3x^2/(3y^2 - 4),$$
   $y(1) = 0$, and determine the interval in which the solution is valid. (*Hint:* To find the interval of definition, look for points where the integral curve has a vertical tangent.)
7. (p. 24 # 26) Consider the initial value problem
$$y' + \frac{2}{3}y = 1 - \frac{1}{2}t,$$
   $y(0) = y_0$. Find the value of y_0 for which the solution touches, but does not cross, the t-axis.
8. (p. 25 # 32) *Variation of Parameters.* Consider the following method of solving the general linear equation of first order:
$$y' + p(t)y = g(t).$$
   (a) If $g(t)$ is identically zero, show that the solution is
$$y = A \exp\left[-\int p(t)dt\right],$$

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where A is a constant.
(b) If $g(t)$ is not identically zero, assume that the solution is of the form

$$y = A(t)\exp\left[-\int p(t)dt\right], \tag{1}$$

where A is now a function of t. By substituting for y in the given differential equation, show that $A(t)$ must satisfy the condition

$$A'(t) = g(t)\exp\left[\int p(t)dt\right]. \tag{2}$$

(c) Find $A(t)$ from (2). Then substitute for $A(t)$ in (1) and determine y. Verify that the solution obtained in this manner agrees with the solution given by integrating factor.
(d) (p. 25 # 34) Use this method to solve $y' + y/t = 3\cos 2t$, $t > 0$.