18.014 QUIZ III

You may use a crib sheet (one side of an 8 1/2 by 11 sheet of papers, but no calculator. Justify all steps — if in doubt, please ask. Time: 50 minutes.

0. (1 point) Write your name and your recitation instructor's name on the first page of your solutions.

1. (27 points) Evaluate:

(a) $\int \frac{dx}{x^3+x^2}$.

(b) $\int_e^{e^2} \frac{dx}{x \ln^2 x}$.

(c) $\int \frac{x^3 dx}{\sqrt{1-x^2}}$.

2. (15 points) Define $\lim_{x\to+\infty} f(x) = 3$ and $\lim_{t\to 0^+} f(1/t) = 3$. Explain why one equality is true if the other is.

3. (18 points) Suppose you use the first two nonzero terms of the Taylor polynomial for $\cos x$ to compute the integral

$$\int_0^{1/2} \cos(x^2)dx.$$

(a) What answer do you get? (Leave as a sum of fractions.)
(b) Is your answer greater or less than the actual value?
(c) Obtain an upper bound on the error. (Leave in terms of fractions.)

4. (12 points) There is a positive integer m such that

$$\lim_{x\to 0} \frac{\sin(2x^3) - 2x^3}{x^m}$$

is finite and nonzero. What is m, and what is the limit L?

5. (27 points) Evaluate:

(a) $\lim_{x\to 0} \frac{\sin^2(ax)}{1-\cos(bx)}$.

(b) $\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

(c) $\lim_{x\to 0^+} \frac{e^{-1/x}}{x}$.

GOOD LUCK!

Date: Fall 2000.